CBCS SCHEME

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17MAT41

Fourth Semester B.E. Degree Examination, Feb./Mar. 2022
Engineering Mathematics – IV

Time: 3 hrs. Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Find the Taylor series method, the value of y at x = 0.1 to five decimal places from $\frac{dy}{dx} = x^2y - 1$, y(0) = 1. Consider upto 4[th] degree terms. **(06 Marks)**

b. Solve $\frac{dy}{dx} = \frac{y-x}{y+x}$ with y(0) = 1 and hence find y(0.1) by taking one step using Runge-Kutta method of fourth order. **(07 Marks)**

c. Given $\frac{dy}{dx} = \frac{x+y}{2}$, given that y(0) = 2, y(0.5) = 2.636, y(1) = 3.595, y(1.5) = 4.968 then find the value of y at x = 2 using Milne's method. **(07 Marks)**

OR

2 a. Using modified Euler's method, solve $\frac{dy}{dx} = x + |\sqrt{y}|$ with y(0) = 1 and hence find y(0.2) with h = 0.2. Modify the solution twice. **(06 Marks)**

b. Use fourth order Runge-Kutta method to find y(0.2), given $\frac{dy}{dx} = 3x + y$, y(0) = 1. **(07 Marks)**

c. Find y at x = 0.4 given $\frac{dy}{dx} + y + xy^2 = 0$ at $y_0 = 1$, $y_1 = 0.9008$, $y_2 = 0.8066$, $y_3 = 0.722$ taking h = 0.1 using Adams-Bashforth method. **(07 Marks)**

Module-2

3 a. Given $\frac{d^2y}{dx^2} = x\left(\frac{dy}{dx}\right)^2 - y^2$. Find y at x = 0.2. Correct to four decimal places, given y = 1 and y′ = 0 when x = 0 using Runge-Kutta method. **(06 Marks)**

b. If α and β are two distinct roots of $J_n(x) = 0$ then prove that $\int_0^1 xJ_n(\alpha x)J_n(\beta x) = 0$ if $\alpha \neq \beta$. **(07 Marks)**

c. Show that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}}\cos x$. **(07 Marks)**

OR

4 a. Given $\frac{d^2y}{dx^2} = 1 + \frac{dy}{dx}$, y(0) = 1, y′(0) = 1, compute y(0.4) for the following data, using Milne's predictor-corrector method.
y(0.1) = 1.1103, y(0.2) = 1.2427, y(0.3) = 1.399
y′(0.1) = 1.2103, y′(0.2) = 1.4427, y′(0.3) = 1.699 **(06 Marks)**

b. Express $x^3 + 2x^2 - x - 3$ in terms of Legendre polynomial. **(07 Marks)**

c. Derive Rodrigue's formula $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}[(x^2 - 1)^n]$ **(07 Marks)**

Module-3

5 a. State and prove Cauchy-Rieman equation in Cartesian form. **(06 Marks)**

b. Evaluate $\int_C \frac{e^{2z}}{(z+2)(z+4)(z+7)} . dz$ where C is the circle $|z| = 3$ using Cauchy's residue theorem. **(07 Marks)**

c. Discuss the transformation $W = e^Z$. **(07 Marks)**

OR

6 a. Prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ **(06 Marks)**

b. State and prove Cauchy's integral formula. **(07 Marks)**

c. Find bilinear transformation which maps $Z = i, 1, -1$ onto $W = 1, 0, \infty$ **(07 Marks)**

Module-4

7 a. A random variable X has the following probability function for various values of x:

X (= xi)	0	1	2	3	4	5	6	7
P(x)	0	K	2K	2K	3K	K^2	$2K^2$	$7K^2$+K

Find: (i) The value of K (ii) $P(x < 6)$ (iii) $P(x \geq 6)$ **(06 Marks)**

b. Derive mean and variance of the binomial distribution. **(07 Marks)**

c. The joint probability distribution of two random variables X and Y as follows:

X \ Y	–4	2	7
1	1/8	1/4	1/8
5	1/4	1/8	1/8

Determine: (i) Marginal distribution of X and Y (ii) Covariance of X and Y (iii) Correlation of X and Y **(07 Marks)**

OR

8 a. In a certain factory turing out razor blades, there is a small chance of 0.002 for a blade to be defective. The blades are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing: (i) no defective (ii) one defective (iii) two defective blades, in a consignment of 10,000 packets. **(06 Marks)**

b. In an examination 7% of students score less than 35% marks and 89% of students score less than 60% marks. Find the mean and standard deviation if the marks are normally distributed. Given $p(0 < z < 1.2263) = 0.39$ and $p(0 < z < 1.4757) = 0.43$. **(07 Marks)**

c. Given:

X \ Y	0	1	2	3
0	0	1/8	1/4	1/8
1	1/8	1/4	1/8	0

Find : (i) Marginal distribution of X and Y (ii) E[X], E[Y], E[XY] **(07 Marks)**

Module-5

9 a. Define the terms:
  (i) Null hypothesis
  (ii) Confidence interval
  (iii) Type-I and Type-II errors **(06 Marks)**

b. A certain stimulus administered to each of the 12 patients resulted in the following change in the blood pressure 5, 3, 8, –1, 3, 0, 6, –2, 1, 5, 0, 4. Can it be concluded that the stimulus will increase the blood pressure ($t_{0.05}$ for 11 d.f is 2.201) **(07 Marks)**

c. Given the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$. Find the fixed probability vector. **(07 Marks)**

OR

10 a. A die thrown 9000 times and a thrown of 3 or 4 was observed 3240 times. Is it reasonable to think that the die is an unbiased one? **(06 Marks)**

b. Four coins are tossed 100 times and the following results were obtained:

Number of Heads	0	1	2	3	4
Frequency	5	29	36	25	5

Fit a binomial distribution for the data and test the goodness of fit [$\chi^2_{0.05} = 9.49$ for 4 d.f]. **(07 Marks)**

c. Every year, a man trades for his car for a new car. If he has Maruti, he trade it for a Tata. If he has a Tata, he trade it for a Honda. However, if he has a Honda, he is just as likely to trade it for a new Honda as to trade it for a Maruti or a Tata. In 2016, he bought his first car which was a Honda. Find the probability that he has (i) 2018 Tata (ii) 2018 Honda (iii) 2018 Maruti. **(07 Marks)**

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